# Discussion for midterm 2 Worksheet Some past exam problems 

Date: 11/3/2021
MATH 53 Multivariable Calculus

1. Find the maximum and minimum values of the function $(x+1)^{2}+y^{2}$ on the ellipse $x^{2}+y^{2} / 4=$ 1 , and say at what points these values occur.
2. Find the area of the region enclosed by the curve $x^{2}+x y+y^{2}=1$. (Hint: use the substitution $x=u+v \sqrt{3}, y=u-v \sqrt{3}$.)
3. Compute $\int_{C} \vec{F} \cdot d \vec{r}$, where $C$ is the unit circle oriented counterclockwise, and $\vec{F}$ is the vector field

$$
\vec{F}=\left(-y^{3}+\sin (\sin x), x^{3}+\sin (\sin y)\right) .
$$

4. Compute

$$
\int_{-2}^{2} \int_{y^{2}}^{4} y \sin \left(x^{2}\right) d x d y
$$

5. Calculate the volume of the region consisting of all points that are inside the sphere $x^{2}+y^{2}+$ $z^{2}=4$, below the cone $z=\sqrt{x^{2}+y^{2}}$, and above the cone $z=-\sqrt{x^{2}+y^{2}}$.

Note: These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.

